

# A Study of the $\pi B \rightarrow Y K$ reactions for Kaon Production in Heavy Ion Collisions\*

K. Tsushima<sup>1†</sup> S. W. Huang<sup>2‡</sup> and Amand Faessler<sup>2</sup>

<sup>1</sup>Department of Physics and Mathematical Physics,

The University of Adelaide, Australia 5005

<sup>2</sup>Institut für Theoretische Physik, Universität Tübingen

Auf der Morgenstelle 14, 72076 Tübingen, F. R. Germany

February 9, 2008

## Abstract

Parametrizations of total cross sections sufficient for all channels of the  $\pi B \rightarrow Y K$  reactions are completed using a resonance model. As well as discussing the  $\pi N \rightarrow \Lambda K$  reactions, which were not presented in our previous publications, we present the differential cross section for  $\pi N \rightarrow \Lambda K$ . This report also aims at presenting supplementary discussions to our previous work.

One of the main goals of studying heavy ion collisions is to determine the equation of state (EOS) of nuclear matter. Because positive kaons ( $K^+$ ) have a long mean free path inside the nucleus they are suggested as a good probe for the reactions occurring in the central region of the collisions [1]. Indeed, theoretical studies show that the kaons produced in heavy ion collisions are sensitive to the EOS [2, 3].

Although one can point out many important ingredients for the theoretical investigations of kaon production in heavy ion collisions, the discussions presented here are concerned with the elementary kaon production cross sections necessary for microscopic calculations.

One of the purposes of this report is to complete the parametrizations of total cross sections sufficient for all channels of the  $\pi B \rightarrow Y K$  reactions by a resonance model [7, 8] ( $B = N, \Delta$  and  $Y = \Lambda, \Sigma$ ). The results for the  $\pi N \rightarrow \Lambda K$  reactions which were not given in our previous publications [7, 8] are given. Furthermore, supplementary discussions to our previous work are presented.

---

\*Talk given at Joint Japan-Australia Workshop on Quarks, Hadrons and Nuclei, Adelaide, South Australia, November 15-24, 1995 (To appear in the conference proceedings)

ADP-96-4/T209

<sup>†</sup>Supported by Australian Research Council

<sup>‡</sup>Supported by GSI under contract No. Tü Fäs T

The microscopic transport models [4] used for the calculations of kaon production in heavy ion collisions contain the following processes as the main collision terms:  $B_1 B_2 \rightarrow B_1 B_2$ ,  $NN \rightarrow NN\pi$ ,  $NN \leftrightarrow N\Delta$ ,  $\pi N \leftrightarrow \Delta$ . Kaons are produced through the  $B_1 B_2 \rightarrow B_3 Y K$  and  $\pi B \rightarrow Y K$  reactions.

For a given impact parameter  $b$ , the Lorentz-invariant differential kaon multiplicity in the microscopic calculations is given by

$$\begin{aligned} E \frac{d^3 N(b)}{d^3 p} &= \sum_{B_1 B_2} \int \left( E' \frac{d^3 \sigma_{B_1 B_2 \rightarrow B_3 Y K}(\sqrt{s_{B_1 B_2}})}{d^3 p'} / \sigma_{B_1 B_2}^{tot}(\sqrt{s_{B_1 B_2}}) \right) [1 - f(\mathbf{r}, \mathbf{p}, t)] \frac{d\Omega_{3Y}}{4\pi} \\ &+ \sum_{\pi B} E'' \frac{d^3 \sigma_{\pi B \rightarrow Y K}(\sqrt{s_{\pi B}})}{d^3 p''} / \sigma_{\pi B}^{tot}(\sqrt{s_{\pi B}}). \end{aligned} \quad (1)$$

Here the primed and the double-primed quantities are in the center-of-momentum (c.m.) frames of the two colliding baryons ( $B_1 B_2$ ) and pion-baryon ( $\pi B$ ), respectively, while the unprimed quantities are those in the c.m. of the two nuclei.  $\sigma_{B_1 B_2}^{tot}(\sqrt{s_{B_1 B_2}})$  and  $\sigma_{\pi B}^{tot}(\sqrt{s_{\pi B}})$  are the total cross sections as functions of the respective c.m. energies  $\sqrt{s_{B_1 B_2}}$  and  $\sqrt{s_{\pi B}}$ . The factor  $[1 - f(\mathbf{r}, \mathbf{p}, t)]$  stands for the Pauli blocking effects for the final baryon  $B_3$ , and  $\Omega_{3Y}$  is the solid angle of the relative momentum between the final baryon  $B_3$  and hyperon  $Y$ . The Lorentz-invariant double differential kaon production cross section is obtained by integrating the kaon multiplicity Eq. (1) over the impact parameter  $b$  multiplied by the factor  $2\pi b$ . Eq.(1) shows that the elementary kaon production cross sections are directly related to the differential kaon yields. Thus, it is important to use correct elementary kaon production cross sections for the microscopic investigations of kaon production in heavy ion collisions.

The elementary kaon production cross sections used for the microscopic calculations are usually taken from Randrup and Ko [5] in baryon-baryon collisions. In pion-*nucleon* collisions they are usually taken from Cugnon and Lombard [6]. Our interest at the moment is with the latter case. In the work of Cugnon and Lombard [6] total cross sections are parametrized by introducing an isospin-averaging procedure, and by using the limited experimental data assuming proton and neutron number ( $N = Z$ ) symmetry. This means that the differential cross sections must be investigated using a more general procedure. Furthermore, the reactions  $\pi\Delta \rightarrow YK$  cannot be investigated because no experimental data are available. One of the motivations for this work is to study both the total and differential cross sections  $\pi B \rightarrow YK$  based on a microscopic model. Below, we give a theoretical description of the  $\pi N \rightarrow \Lambda K$  reactions which will be our main concern later.

The experimental data [9] show that three resonances  $N(1650)(\frac{1}{2}^-)$ ,  $N(1710)(\frac{1}{2}^+)$  and  $N(1720)(\frac{3}{2}^+)$  make contributions to the  $\pi N \rightarrow \Lambda K$  reactions. In addition,  $K^*(892)$  exchange is included as an effective process which also simulates the other heavier  $K^*$  meson effects. The relevant processes are depicted in Fig. 1. The interaction Lagrangians used are:

$$\mathcal{L}_{\pi N N(1650)} = -g_{\pi N N(1650)} \left( \bar{N}(1650) \vec{\tau} N \cdot \vec{\phi} + \bar{N} \vec{\tau} N(1650) \cdot \vec{\phi} \right), \quad (2)$$

$$\mathcal{L}_{\pi NN(1710)} = -ig_{\pi NN(1710)} \left( \bar{N}(1710)\gamma_5 \vec{\tau} N \cdot \vec{\phi} + \bar{N} \vec{\tau} \gamma_5 N(1710) \cdot \vec{\phi} \right), \quad (3)$$

$$\mathcal{L}_{\pi NN(1720)} = \frac{g_{\pi NN(1720)}}{m_\pi} \left( \bar{N}^\mu(1720) \vec{\tau} N \cdot \partial_\mu \vec{\phi} + \bar{N} \vec{\tau} N^\mu(1720) \cdot \partial_\mu \vec{\phi} \right), \quad (4)$$

$$\mathcal{L}_{K\Lambda N(1650)} = -g_{K\Lambda N(1650)} \left( \bar{N}(1650)\Lambda K + \bar{K} \bar{\Lambda} N(1650) \right), \quad (5)$$

$$\mathcal{L}_{K\Lambda N(1710)} = -ig_{K\Lambda N(1710)} \left( \bar{N}(1710)\gamma_5 \Lambda K + \bar{K} \bar{\Lambda} \gamma_5 N(1710) \right), \quad (6)$$

$$\mathcal{L}_{K\Lambda N(1720)} = \frac{g_{K\Lambda N(1720)}}{m_K} \left( \bar{N}^\mu(1720) \Lambda \partial_\mu K + (\partial_\mu \bar{K}) \bar{\Lambda} N^\mu(1720) \right), \quad (7)$$

$$\mathcal{L}_{K^*(892)\Lambda N} = -g_{K^*(892)\Lambda N} (\bar{N} \gamma^\mu \Lambda K_\mu^*(892) + \bar{K}^*_{\mu}(892) \bar{\Lambda} \gamma^\mu N), \quad (8)$$

$$\mathcal{L}_{K^*(892)K\pi} = if_{K^*(892)K\pi} \left( \bar{K} \vec{\tau} K_\mu^*(892) \cdot \partial^\mu \vec{\phi} - (\partial^\mu \bar{K}) \vec{\tau} K_\mu^*(892) \cdot \vec{\phi} \right) + \text{h.c.} \quad (9)$$

The amplitudes are given by:  $\mathcal{M}_{\pi^0 p \rightarrow \Lambda K^+} = -\mathcal{M}_{\pi^0 n \rightarrow \Lambda K^0} = \frac{1}{\sqrt{2}} \mathcal{M}_{\pi^+ n \rightarrow \Lambda K^+} = \frac{1}{\sqrt{2}} \mathcal{M}_{\pi^- p \rightarrow \Lambda K^0} = \mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c + \mathcal{M}_d$ , where the amplitudes  $\mathcal{M}_a$ ,  $\mathcal{M}_b$ ,  $\mathcal{M}_c$  and  $\mathcal{M}_d$  correspond to the diagrams (a), (b), (c) and (d), respectively in Fig. 1.

To carry the calculations further, form factors are introduced which reflect the finite size of the hadrons. Those form factors are carried by each vertex. For the meson-baryon-(baryon resonance) vertex, the following form factor is used:

$$F(\vec{q}) = \frac{\Lambda_C^2}{\Lambda_C^2 + \vec{q}^2}, \quad (10)$$

where  $\vec{q}$  is the meson momentum, and  $\Lambda_C$  is the cut-off parameter. On the other hand, for the  $K^*(892)$ - $K$ - $\pi$  vertex, the form factor studied in Ref. [10] is used:

$$F_{K^*(892)K\pi}(|\frac{1}{2}(\vec{p}_K - \vec{p}_\pi)|) = C |\frac{1}{2}(\vec{p}_K - \vec{p}_\pi)| \exp\left(-\beta |\frac{1}{2}(\vec{p}_K - \vec{p}_\pi)|^2\right). \quad (11)$$

Before discussing the results, the model parameters need to be specified. The cut-off parameter  $\Lambda_C$  appearing in Eq. (10) is  $\Lambda_C = 0.8$  GeV for all meson-baryon-(baryon resonance) vertices. The values obtained for the coupling constants with this cut-off value are given in Table 1. The fitted value for  $g_{K^*(892)\Lambda N}$  is  $g_{K^*(892)\Lambda N} = 0.45$ . The other parameters  $C$  and  $\beta$  appearing in Eq. (11) are  $C = 2.72$  fm and  $\beta = 8.88 \times 10^{-3}$  fm<sup>2</sup> obtained in Ref. [10].

Here, it is appropriate to discuss  $K^*(892)$  exchange. Our calculations were also performed with the inclusion of the tensor coupling interaction. However, it was found that the results show similar dependence on both c.m. energy and angle (or  $\cos \theta_{c.m.}$ ) to those results calculated with the inclusion of the vector coupling interaction alone. Thus, for the present purposes it is enough to include only the vector coupling interaction.

The energy dependence of the total cross sections  $\pi^- p \rightarrow \Lambda K^0$  is shown in Fig. 2. Figures (a) and (b) correspond to cases without and with the inclusion of interference terms, respectively. The sign combination among the interference terms in calculation (b) is selected in such a way that *both* total and differential cross sections are reproduced simultaneously. Mere inclusion of the single resonance or the  $K^*$  exchange alone cannot reproduce the energy dependence of the total cross section data.

Next, the differential cross sections  $\pi^- p \rightarrow \Lambda K^0$  are shown in Fig. 3. Figures (a), (b) and (c) correspond to the pion beam momenta 0.980 GeV/c, 1.13 GeV/c and 1.455 GeV/c, respectively. The general trends are reproduced, but the details are not yet satisfactory.

In Fig. 4, we give the energy dependence of the total cross sections  $\pi N \rightarrow \Sigma K$ . In Ref. [7], the  $\Delta(1920)$  resonance was treated as an effective resonance which simulates other  $\Delta$  resonance effects around the mass region 1.9 GeV. The two coupling constants  $g_{K\Sigma\Delta(1920)}$  and  $g_{\pi N\Delta(1920)}$  were scaled in Ref. [7]. The results obtained by using these scaled coupling constants are denoted by *set 1*. A more quantitative discussion of this scaling will be made below.

The experimental data [14] show that there are six  $\Delta$  resonances which make contributions to the  $\pi N \rightarrow \Sigma K$  reactions around the mass region 1.9 GeV. (See Table 2.) In order to understand the scaling factor quantitatively, we compare:

$$\frac{\text{the contribution of } \Delta(1920) \text{ to } \pi N \rightarrow \Sigma K}{\text{all } \Delta s'(\text{masses around 1.9 GeV}) \text{ contribution to } \pi N \rightarrow \Sigma K} = \frac{10.4}{37.37} = 0.278,$$

$$\sqrt{\frac{g_{K\Sigma\Delta(1920)}^2 g_{\pi N\Delta(1920)}^2 (\text{from branching ratio})}{g_{K\Sigma\Delta(1920)}^2 g_{\pi N\Delta(1920)}^2 (\text{scaled})}} = \sqrt{\frac{1.11 \times 0.417}{3.83 \times 1.44}} = 0.289.$$

This comparison shows that the scaling factor ( $= 1.861$ ) for each coupling constant  $g_{K\Sigma\Delta(1920)}$  and  $g_{\pi N\Delta(1920)}$  is consistent with the total branching ratio obtained by summing these  $\Delta$  resonance contributions. However, because of this effective description, the differential cross sections cannot be reproduced well. A more accurate determination of the branching ratios for these  $\Delta$  resonances is necessary.

Here, the  $\pi\Delta \rightarrow YK$  reactions should also be mentioned. The parametrizations and figures given in Ref. [8] were obtained by using the lower values of the branching ratios for the resonances to decay to  $\pi\Delta$ . The parametrizations obtained by using the averaged values as given in Tables 1 and 2 of Ref. [8] will be given later. The difference between the previous parametrizations and those to be given later is that in the latter the change in the multiplication factor is large. However, the second term of the parametrization for the  $\pi^+\Delta^0 \rightarrow \Sigma^0 K^+$  total cross section remains the same as before. As for the differential cross sections  $\pi\Delta \rightarrow YK$ , they are almost constant as a function of  $\cos\theta$  in the c.m. frame for the beam energies for which calculations were made.

Finally, the parametrizations of the total cross sections sufficient for all channels of the  $\pi B \rightarrow YK$  reactions in units of mb are given by:

$$\begin{aligned} \sigma(\pi^- p \rightarrow \Lambda K^0) &= \frac{0.007665(\sqrt{s} - 1.613)^{0.1341}}{(\sqrt{s} - 1.720)^2 + 0.007826}, \\ \sigma(\pi^+ p \rightarrow \Sigma^+ K^+) &= \frac{0.03591(\sqrt{s} - 1.688)^{0.9541}}{(\sqrt{s} - 1.890)^2 + 0.01548} + \frac{0.1594(\sqrt{s} - 1.688)^{0.01056}}{(\sqrt{s} - 3.000)^2 + 0.9412}, \\ \sigma(\pi^- p \rightarrow \Sigma^- K^+) &= \frac{0.009803(\sqrt{s} - 1.688)^{0.6021}}{(\sqrt{s} - 1.742)^2 + 0.006583} + \frac{0.006521(\sqrt{s} - 1.688)^{1.4728}}{(\sqrt{s} - 1.940)^2 + 0.006248}, \end{aligned}$$

$$\begin{aligned}
\sigma(\pi^+ n \rightarrow \Sigma^0 K^+) &= \sigma(\pi^0 n \rightarrow \Sigma^- K^+) = \frac{0.05014(\sqrt{s} - 1.688)^{1.2878}}{(\sqrt{s} - 1.730)^2 + 0.006455}, \\
\sigma(\pi^0 p \rightarrow \Sigma^0 K^+) &= \frac{0.003978(\sqrt{s} - 1.688)^{0.5848}}{(\sqrt{s} - 1.740)^2 + 0.006670} + \frac{0.04709(\sqrt{s} - 1.688)^{2.1650}}{(\sqrt{s} - 1.905)^2 + 0.006358}, \\
\sigma(\pi^- \Delta^{++} \rightarrow \Lambda K^+) &= \frac{0.009883(\sqrt{s} - 1.613)^{0.7866}}{(\sqrt{s} - 1.720)^2 + 0.004852}, \\
\sigma(\pi^- \Delta^{++} \rightarrow \Sigma^0 K^+) &= \frac{0.007448(\sqrt{s} - 1.688)^{0.7785}}{(\sqrt{s} - 1.725)^2 + 0.008147}, \\
\sigma(\pi^0 \Delta^0 \rightarrow \Sigma^- K^+) &= \frac{0.01052(\sqrt{s} - 1.688)^{0.8140}}{(\sqrt{s} - 1.725)^2 + 0.007713}, \\
\sigma(\pi^+ \Delta^0 \rightarrow \Sigma^0 K^+) &= \frac{0.003100(\sqrt{s} - 1.688)^{0.9853}}{(\sqrt{s} - 1.725)^2 + 0.005414} + \frac{0.3179(\sqrt{s} - 1.688)^{0.9025}}{(\sqrt{s} - 2.675)^2 + 44.88}, \\
\sigma(\pi^+ \Delta^- \rightarrow \Sigma^- K^+) &= \frac{0.02629(\sqrt{s} - 1.688)^{1.2078}}{(\sqrt{s} - 1.725)^2 + 0.003777},
\end{aligned}$$

where all the parametrizations given above should be understood to be zero below threshold. Furthermore, the parametrizations for the  $\pi\Delta \rightarrow YK$  reactions are obtained without the inclusion of interference terms. The above are the completion of the parametrizations for the total cross sections  $\pi B \rightarrow YK$ . Parametrizations for the other channels can be obtained by multiplying the relevant constant factors arising from isospin space [7, 8].

The next task is to investigate the  $B_1 B_2 \rightarrow B_3 YK$  reactions by using the same resonance model, where most of the model parameters have already been fixed by this investigation. This program is now in progress.

**Acknowledgement:** One of the authors (K.T.) expresses his thanks to Prof. A.W. Thomas and to Drs. A.G. Williams and K. Saito who organized “Quarks, Hadron and Nuclei Workshop” held in Adelaide for the opportunity to report the results mentioned here.

## References

- [1] S. Schnetzer et al., Phys. Rev. Lett. 49 (1982) 989; S. Nagamiya, Phys. Rev. Lett. 49 (1982) 1383.
- [2] J. Aichelin and C. M. Ko, Phys. Rev. Lett. 55 (1985) 2661.
- [3] A. Lang et al., Nucl. Phys. A 541 (1992) 507; S. W. Huang et al., Phys. Lett. B 298 (1993) 41; G. Hartnack et al., Nucl. Phys. A580 (1994) 643.
- [4] W. Cassing et al., Phys. Rep. 188 (1990) 363; G. Q. Li, A. Faessler and S. W. Huang, Prog. Part. and Nucl. Phys. 31 (1993) 159; J. Aichelin, Phys. Rep. 202 (1991) 235.

- [5] J. Randrup and C. M. Ko, Nucl. Phys. A 343 (1980) 519; Nucl. Phys. A 411 (1983) 537.
- [6] J. Cugnon and R. M. Lombard, Nucl. Phys. A 422 (1984) 635.
- [7] K. Tsushima, S. W. Huang, Amand Faessler, Phys. Lett. B 337 (1994) 245.
- [8] K. Tsushima, S. W. Huang and Amand Faessler, J. Phys. G. 21 (1995) 33.
- [9] Particle Data Group, Phys. Rev. D 45 (1992); Phys. Lett. B 239 (1990) 1.
- [10] C. Gobbi, F. Iachello and D. Kusnezov, Phys. Rev. D 50 (1994) 2048.
- [11] A. Baldini et al., *Landolt-Börnstein, Numerical Data and Functional Relationships in Science and Technology*, Vol. 12, ed. by H. Schopper, (Springer-Verlag, 1988).
- [12] T. M. Knasel et al., Phys. Rev. D 11 (1975) 1.
- [13] D. H. Saxon et al., Nucl. Phys. B162 (1980) 522.
- [14] Particle Data Group, Phys. Rev. D 50 (1994) 1173-1826.

### Captions

**Fig. 1:** The processes contributing to the  $\pi N \rightarrow \Lambda K$  reactions. The diagrams correspond to (a) :  $N(1650) I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ , (b) :  $N(1710) \frac{1}{2}(\frac{1}{2}^+)$ , (c) :  $N(1720) \frac{1}{2}(\frac{3}{2}^+)$  s-channels and (d) :  $K^*(892)$ -exchange, respectively.

**Fig. 2:** The total cross sections  $\pi^- p \rightarrow \Lambda K^0$ . The experimental data are taken from Ref. [11]. The results are: (a)-without and (b)-with the inclusion of interference terms, respectively.

**Fig. 3:** The differential cross sections  $\pi^- p \rightarrow \Lambda K^0$  in c.m. frame. (a), (b) and (c) correspond to the pion beam momenta 0.980 GeV/c ( $\sqrt{s} = 1.66$  GeV), 1.13 GeV/c ( $\sqrt{s} = 1.742$  GeV) and 1.455 GeV/c ( $\sqrt{s} = 1.908$  GeV), respectively. For (a) and (b), the experimental data are taken from Ref. [12], while for (c) they are taken from Ref. [13].

**Fig. 4:** The total sections for the reactions (a):  $\pi^+ p \rightarrow \Sigma^+ K^+$ , (b):  $\pi^- p \rightarrow \Sigma^- K^+$  and (c):  $\pi^+ n \rightarrow \Sigma^0 K^+$  and  $\pi^0 n \rightarrow \Sigma^- K^+$ , respectively. The notation *set 1* in these figures indicates the results obtained by using the scaled values for  $g_{K\Sigma\Delta(1920)}$  and  $g_{\pi N\Delta(1920)}$ . For further explanations, see Ref. [7].

Table 1: The calculated coupling constants and the experimental branching ratios.

$B^*(\text{resonance})$	$\Gamma^{full}(MeV)$	$\Gamma_{N\pi}(\%)$	$g_{\pi NB^*}^2$	$\Gamma_{\Lambda K}(\%)$	$g_{K\Lambda B^*}^2$
$N(1650)$	150	70.0	1.41	7.0	$6.40 \times 10^{-1}$
$N(1710)$	100	15.0	2.57	15.0	$4.74 \times 10^{+1}$
$N(1720)$	150	15.0	$5.27 \times 10^{-2}$	6.5	3.91
$f_{K^*(892)K\pi}^2$			$g_{K^*(892)\Lambda N}^2$		
$6.89 \times 10^{-1}$			$2.03 \times 10^{-1}$		
$(\Gamma=50 \text{ MeV}, \quad \Gamma_{K\pi} = 100\%)$					

Table 2: Contribution of  $\Delta(1920)$  to the  $\pi N \rightarrow \Sigma K$  reactions compared with other possible  $\Delta$  resonances between 1900 and 1940 MeV [14]. The values in column four are all upper values.

$\Delta^*(\text{resonance})$	$\Gamma_{total}(MeV)$	$(\Gamma_{\pi N \Delta^*} \Gamma_{K \Sigma \Delta^*})^{1/2} / \Gamma_{total}$	$(\Gamma_{\pi N \Delta^*} \Gamma_{K \Sigma \Delta^*})^{1/2} (MeV)$
$\Delta(1900)$	200	$< 0.03$	6.00
$\Delta(1905)$	350	$  0.015  $	5.25
$\Delta(1910)$	250	$< 0.03$	7.50
$\Delta(1920)$	200	$  0.052  $	10.4
$\Delta(1930)$	350	$< 0.015$	5.25
$\Delta(1940)$	198.4	$< 0.015$	2.98

This figure "fig1-1.png" is available in "png" format from:

<http://arXiv.org/ps/nucl-th/9602005v2>



This figure "fig1-2.png" is available in "png" format from:

<http://arXiv.org/ps/nucl-th/9602005v2>

This figure "fig1-3.png" is available in "png" format from:

<http://arXiv.org/ps/nucl-th/9602005v2>

This figure "fig1-4.png" is available in "png" format from:

<http://arXiv.org/ps/nucl-th/9602005v2>